III. The Capital Asset Pricing Model

- Most of professional investors try to identify mispriced securities. They buy underpriced assets and short sell overpriced ones.
- To find a mispriced security an investor needs a model that provides a benchmark price of a given security. An investor comparers a market price (or his estimate of an expected return) with a benchmark price (or benchmark expected return) to see if security is mispriced.
- Now our goal is to determine benchmark expected returns of risky securities. Often, these returns are found in a general equilibrium framework. This framework relies on a supply–demand analysis. Therefore, the benchmark returns are also called "equilibrium", "efficient" or "fair" returns.
- The Capital Asset Pricing Model (CAPM) is a model that predicts expected returns on risky assets assuming that all investors are rational and use optimal trading strategies that provide the highest expected utilities
- CAPM assumes that supply of stock shares is exogenously fixed while demand for shares comes from investors. In equilibrium supply should be equal to demand which is possible only at a certain price of a stock share. We will find this price (and return) by considering an equilibrium in the stock market. This price is used as a benchmark to find mispriced securities
- The model makes a number of strong assumptions but allows us to gain a powerful insight into the nature of equilibrium in security markets and the properties of security returns

Assumptions of CAPM:

A1 There are many investors, each with an endowment (wealth) that is small relative to the total endowment of all investors

- This assumption is equivalent to perfect competition in microeconomics
- It implies that individual investors are price takers

A2 Single-period investment horizon

- That is, investors trade only once and receive the proceeds from their trades at the end of their investment horizon. An investment horizon is the same for all investors and it could be a few weeks, a few months or even many years

A3 Investments are limited to publicly traded financial assets (stocks and bonds)

- Rules out, among other things, investments in human capital (education) and private enterprizes
- Implies that investors may borrow or lend at a fixed, risk-free rate

A4 No taxes and transaction costs

- Both are important in real world

A5 Investors are rational mean–variance optimizers, meaning that they all use Markowitz portfolio selection model

A6 Homogeneous expectations

- All investors have the same beliefs about future returns. Therefore, they use the same input list resulting in the same efficient frontier

- The assumptions above result in the equilibrium with the following properties:

P1 All investors in equilibrium must hold the market portfolio.

- The **market portfolio** (M) is the portfolio that includes all shares of all publicly traded assets.

– The price of the market portfolio is

$$S_M = \sum_{k=1}^{I} N_k S_k,$$

where I is a total number of stocks.

– If all investors use identical Markowitz analysis (A5) on the same universe of securities (A3) for the same time horizon (A2) and use the same input list (A6), then they must all arrive at the same optimal risky portfolio P. P should be the market portfolio.

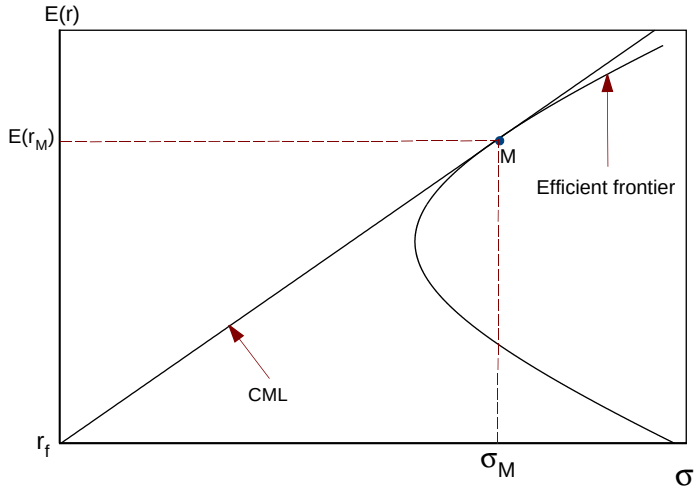
– Let us consider an example illustrating the last statement. Suppose that there are 1,000,000 investors each holding risky portfolio worth $\$1,000,000$. It follows that the value of the market portfolio is $1,000,000 \times \$1,000,000 = \$10^{12}$. Let the weight of stock X in this portfolio be the same for each investor and equal to 1% so that the dollar value allocated to this stock by each investor is $0.01 \times \$1,000,000 = \$10,000$. Suppose the weight of this stock in the market portfolio is 2%. It follows that capitalization of company X is $0.02 \times \$10^{12} = \$2 \times 10^{10}$ while the demand for this stock by investors is $1,000,000 \times \$10,000 = \$1 \times 10^{10}$. Therefore, the market for stock X is not in equilibrium since supply is twice of the demand. The market is in equilibrium only if weight of stock X in the portfolio of each investor is equal to 2%. Hence each investor must hold market portfolio M in equilibrium.

– Investors will differ only in fraction of their complete portfolio invested in the market portfolio, M.

– Because each investor holds a market portfolio, the capital market line (CML) is also the best attainable capital allocation line

P2 Risk premium on the market depends on the average risk aversion of all market participants

$$E(r_M) - r_f = \bar{A}\sigma_M^2$$



where $\bar{A} = \frac{1}{\sum_{i=1}^{N} \frac{W_i}{S_M} \frac{1}{A_i}}$, A_i, W_i are risk aversion and wealth of investor i, and N is a total number of investors in economy

- Question: Why market risk premium is always positive in CAPM?

- Question: In practice, do we always expect market risk premium to be positive?

Example:

Suppose the average risk aversion of market participants is 4 and the variance of return on the market portfolio is 0.0225. If the risk-free rate of return is 0.06, what is the expected return on the market portfolio?

$$E(r_M) = r_f + \bar{A}\sigma_M^2 = 0.06 + 4 \times 0.0225 = 0.15$$

P3 Risk premium on an individual security is a function of its covariance with the market

- This is the most important prediction of CAPM, so let's briefly talk about the intuition behind this prediction;
  - A couple of lectures ago we talked about a total risk of an asset and said that it can be divided into two parts.
  - Question: what were those parts?
  - Answer:
  - So in a well-diversified portfolio we will be left only with the market risk (systematic risk)
  - Because an investor holds the market portfolio, she is not exposed to the nonsystematic risk and does not price it. Therefore investors will be compensated by higher expected risk premium only for bearing the market risk and not the unique risk.
  - Since market portfolio has only systematic risk and it is not correlated with nonsystematic risk, it is natural that the amount of the market risk in a given stock i is proportional to $cov(r_i, r_M)$. It makes even more sense for us to expect that it should be proportional to $cov(r_i, r_M)/\sigma_M^2$.
  - Therefore
    $$E(r_i) - r_f = cov(r_i, r_M)/\sigma_M^2 \times B$$
    where B is some number
  - Now, because $cov(r_i, r_M)/\sigma_M^2$ for the market portfolio is 1 we find that $B = E(r_M) - r_f$ and arrive to

CAPM equation

$$E(r_i) = r_f + \beta_i[E(r_M) - r_f],$$

where

$$\beta_i = \frac{Cov(r_i, r_M)}{\sigma_M^2}$$

- We conclude that the expected return demanded by investors for holding an asset depends on:
  - The time value of money. It is measured by the risk–free rate, r_f, which is the reward for waiting for your money, without taking any risk
  - The risk premium. This is the reward for bearing systematic risk.
- The beta of a stock measures the degree to which the stock moves with the market and the magnitude of those movements. If firm–specific risk is neglected:
  - A beta of one means the stock moves with the market: in other words when the market moves up 10% the stock moves up 10%.

    Example:
  - A stock with a beta between 0 and 1 moves with the market but to a lessor degree. Such a stock is called a conservative investment.

    Example:
  - A stock with a beta greater than 1 is a stock which moves in the same direction as the market but it moves more than the market does. Such a stock is referred to as aggressive.

    Example:
  - A stock which has a negative beta moves in the direction that is opposite to that of the market.

    Example:

Question: Consider the investment horizon of 50 years. Do we know any security that historically had negative β for this horizon?

Practice problem

Suppose that the risk–free rate is 3%, the market risk premium is 7%, and the beta of stock is 1.5. Based on CAPM, what is the expected return on the stock?

$E(r_i) =$

What if the beta is decreased to 0.4?

$E(r_i) =$

- CAPM equation gives the expected return of a stock when it is fairly (efficiently) priced. However, the market allows mispricing of some stocks with respect to CAPM prediction. For such stock

$$E(r_i) = \alpha_i + r_f + \beta_i[E(r_M) - r_f],$$

where α_i is called the stock's alpha.

  - If $\alpha > 0$ then stock is underpriced, if $\alpha < 0$ then stock is overpriced
  - An investor can take advantage of stock mispricing only if its price will be adjusted and follow the CAPM later. That is, all investors have to believe in CAPM and use it for pricing securities

Example

Suppose that the risk–free rate is 5%, the market risk premium is 8%, and the beta of stock is 1.2. The CAPM would predict an expected return

$E(r) =$

If one believed the stock would provide a return of 19%, the implied α would be

$\alpha =$

- If the expected return–beta relationship holds for any individual asset, it must hold for any combination of assets
  - Suppose that portfolio P has weight w_k for stock k $(k = 1, ..., I)$
  - Let us write down the CAPM equation for each stock and multiply equation for each stock by its weight in portfolio P:

$$\begin{aligned} w_1 E(r_1) &= w_1 r_f + w_1 \beta_1 [E(r_M) - r_f] \\ w_2 E(r_2) &= w_2 r_f + w_2 \beta_2 [E(r_M) - r_f] \\ &... \\ w_I E(r_I) &= w_I r_f + w_I \beta_I [E(r_M) - r_f] \end{aligned}$$

  - Summing all equations above results in

$$E(r_P) = r_f + \beta_P [E(r_M) - r_f],$$

    where

$$\begin{aligned} \beta_P &= \sum_k w_k \beta_k = \sum_k w_k \frac{Cov(r_k, r_M)}{\sigma_M^2} = \frac{\sum_k Cov(w_k r_k, r_M)}{\sigma_M^2} \\ &= \frac{Cov(\sum_k w_k r_k, r_M)}{\sigma_M^2} = \frac{Cov(r_P, r_M)}{\sigma_M^2} \end{aligned}$$

    It follows that β_P is the portfolio beta

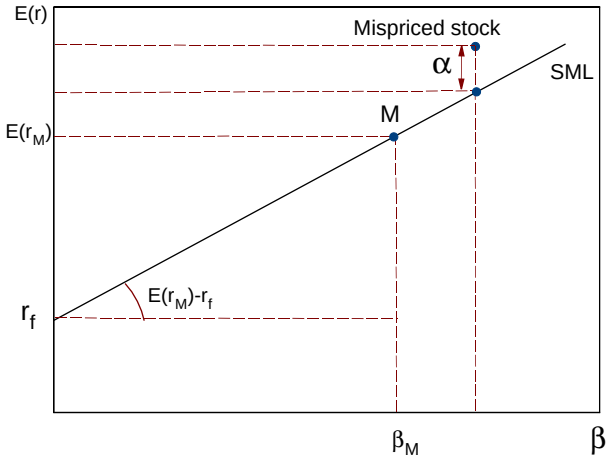
Example

Suppose your portfolio consists of 70% of stock A and 30% of stock B. The beta of stock A is 1.4, the beta of stock B is -0.4. What is the beta of your portfolio?

$\beta_P = w_1 \beta_1 + w_2 \beta_2 =$

The Security Market Line

- The expected return–beta relationship can be depicted graphically as the security market line (SML):



- The slope of the SML is the risk premium of the market portfolio
- "Fairly priced" assets plot exactly on SML. Assets that are mispriced lie away from SML (see the figure above)

Implications from CAPM

- The passive strategy is efficient
  - According to CAPM each investor holds the market portfolio
  - Because the market portfolio does not require any security analysis, investing in it is a passive strategy. Thus, the passive strategy is efficient
- The CAPM provides a benchmark for required returns from investments into financial securities

– To attract investors to actively managed funds, their managers have to provide returns above those of passive funds for a given level of risk

– That is, these managers have to identify mispriced securities (that have $\alpha \neq 0$) to beat the CAPM benchmark and justify a management fee

Example

Suppose that manager's portfolio has $E(r_P) = 11\%$, $\beta_P = 0.8$, while $E(r_M) = 12\%$, $r_f = 6\%$. Is manager's return enough to compensate you for the risk you had to bear?

$$\begin{aligned} \alpha &= \text{Actual expected return-benchmark expected return} \\ &= E(r_P) - [r_f + \beta_P(E(r_M) - r_f)] = 11 - [6 + 0.8(12 - 6)] = 0.2 > 0 \end{aligned}$$

Therefore, the manager is successful in choosing stocks with returns more than adequate to compensate for the level of risk

Practice Problem

A share of stock with a $\beta = 0.7$ now sells for \$43. Investors expect the stock to pay a year–end dividend of \$2 and to sell for \$48 at year–end. The T–bill rate is 4%, and the market return is 14%. Is this stock a good or bad buy?

expected rate of return =

fair expected rate of return =

- The CAPM provides a discount rate for the net present value of a project

– If the NPV from the investment into real assets (production facilities, real estate, land) is positive then undertake the project

Example

You are a consultant for a large manufacturing corporation that is considering a project with the following cash flows (in millions of dollars)

Years from now	CF
0	-23.32
1	5
2	7
3	10
4	10
5	5
6	5

You have just estimated that the project's beta is 1.4. The risk–free rate is 7% and the expected return of the market is 12%

(a) What is the discount rate for this project?

(b) What is the NPV of the project?

(c) What is the highest possible beta estimate for the project before its NPV becomes negative?

Validity of CAPM

- After all the time we have spent on studying CAPM the key question arises: Does CAPM hold empirically?

  - Empirical testing of CAPM is problematic since the model considers expected risk and expected returns while we observe only actual returns and variability. There is also an issue of what should be a proxy for a market portfolio
  - The general relation implied by CAPM that higher expected market risk in security implies a higher expected return of this security has been documented.
  - The central prediction of CAPM that α's of securities are zero's fail empirically. For example, Fama and French (1992) found that an expected return on security depends on the book–to–market ratio and the value of the company
  - The empirical failure of CAPM is due to its strong simplifying assumptions. For example, including production of companies in the model can fix the problem discovered by Fama and French (1992).
  - Despite its limitations, CAPM remains the most popular asset pricing model. The latter is due to its simplicity and appealing economic intuition

Additional Practice Problems

1. Which statement is not true regarding the market portfolio?

A) It includes all publicly traded financial assets.

B) It lies on the efficient frontier.

C) All securities in the market portfolio are held in proportion to their market values.

D) It is the tangency point between the capital market line and the indifference curve.

E) all of the above are true.

2. Your personal opinion is that security X has an expected rate of return of 0.11. It has a beta of 1.5. The risk-free rate is 0.05 and the market expected rate of return is 0.09. According to the Capital Asset Pricing Model, this security is

A) underpriced.

B) overpriced.

C) fairly priced.

D) cannot be determined from data provided.

E) none of the above.

3. According to CAPM, the fair return of the stock is 15%.

(a) What is the stock's beta if the risk–free rate is 3% and the market return is 12%?

(b) What is the standard deviation of the return on the market portfolio if $cov(r_i, r_M) = 0.06$

4. The fair expected return of the stock is 15%. Find the market risk premium if the risk–free rate is 5% and the stock's beta is 0.9. What is the market expected return?